

# CH 7 PERMUTATION AND COMBINATION

## ANSWERS AND EXPLANATIONS

### EXERCISE 1

1. (e)

O, A, E	S	F	T	W	R
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When the vowels are always together, then treat all the vowels as a single letter and then all the letters can be arranged in  $6!$  ways and also all three vowels can be arranged in  $3!$  ways. Hence, required no. of arrangements

$$= 6! \times 3! = 4320.$$

2. (a) Req'd no. of ways =  ${}^7C_4 \times {}^8C_4$

$$= \frac{7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4} \times \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4}$$

$$= 35 \times 70 = 2450$$

3. (b) Treat B and T as a single letter. Then the remaining letters ( $5 + 1 = 6$ ) can be arranged in  $6!$  ways. Since, O is repeated twice, we have to divide by 2 and the B and T letters can be arranged in  $2!$  ways.

Total no. of ways

$$= \frac{6! \times 2!}{2} = 720$$

4. (a) Req'd. number of ways

$$\frac{6!}{2! \times 2!} = \frac{6 \times 5 \times 4 \times 3}{1 \times 2} = 180$$

5. (b)  $27^2 < 765 < 28^2$

$$\therefore \text{required no. of chairs to be excluded} \\ = 765 - 729 = 36$$

6. (a) Req'd. number =  $4! \times 2! = 24 \times 2 = 48$

7. (e) The word SIGNATURE consists of nine letters comprising four vowels (A, E, I and U) and five

consonants (G, N, R, T and S). When the four vowels are considered as one letter, we have six letters which can be arranged in  ${}^6P_6$  ways i.e.  $6!$  ways. Note that the four vowels can be arranged in  $4!$  ways.

Hence required number of words

$$= 6! \times 4!$$

$$= 720 \times 24 = 17280$$

8. (c) Here, 5 men out of 8 men and 6 women out of 10 women can be chosen in

$${}^8C_5 \times {}^{10}C_6 \text{ ways, i.e., } 11760 \text{ ways.}$$

9. (b) We have  $\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30800}{1}$

$$\text{Or } {}^{56}P_{r+6} = 30800 ({}^{54}P_{r+3})$$

$$\text{Or } \frac{(56)!}{(56-r-6)!} = 30800 \times \frac{(54)!}{(54-r-3)!}$$

$$\text{Or } \frac{56 \times 55 \times (54)!}{(50-r)!} = 30800 \times \frac{(54)!}{(51-r)!}$$

$$\text{Or } 56 \times 55 \times (51-r) = 30800$$

$$\text{Or } 51-r = \frac{30800}{56 \times 55} = 10$$

$$\therefore r = 51 - 10 = 41$$

10. (a)  ${}^9P_5 + 5 \cdot {}^9P_4 = {}^{10}P_r$

$$\text{Or } \frac{9!}{(9-5)!} + 5 \cdot \frac{9!}{(9-4)!} = \frac{(10)!}{(10-r)!}$$

$$\text{Or } \frac{9!}{4!} + 5 \cdot \frac{9!}{5!} = \frac{(10)!}{(10-r)!}$$

$$\text{Or } \frac{9!}{4!} + \frac{9!}{4!} = \frac{(10)!}{(10-r)!}$$



$$\text{Or } 2 \times \frac{9!}{4!} = \frac{(10)!}{(10-r)!}$$

$$\text{Or } \frac{(10)!}{5!} = \frac{(10)!}{(10-r)!}$$

$$\Rightarrow 10-r=5 \text{ or } r=5$$

11. (b) The inequality is  ${}^{n+1}C_3 - {}^{n+1}C_2 \leq 100$

We must have  $n+1 \geq 3$  and  $n+1 \geq 2$

$$\Rightarrow n \geq 2 \text{ and } n \geq 1$$

$$\Rightarrow n \geq 2 \text{ and also}$$

$$\frac{(n+1)n(n-1)}{6} - \frac{(n+1)n}{2} \leq 100$$

$$\Rightarrow (n+1)n(n-4) \leq 600$$

By trial the values of  $n$  satisfying this are 2, 3, 4, 5, 6, 7, 8, 9 which are eight in number.

12. (b)  ${}^mC_3 + {}^mC_4 > {}^{m+1}C_3$

$$\Rightarrow {}^{m+1}C_4 > {}^{m+1}C_3$$

$$[\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}]$$

$$\Rightarrow \frac{(m+1)!}{(m-3)!4!} > \frac{(m+1)!}{(m-2)!3!} \Rightarrow m-2 > 4$$

$$\Rightarrow m > 6 \Rightarrow \text{The least value of } m \text{ is } 7.$$

13. (b)  ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$

$$\Rightarrow {}^{39}C_{3r-1} + {}^{39}C_{3r} = {}^{39}C_{r^2-1} + {}^{39}C_{r^2}$$

$$\Rightarrow {}^{40}C_{3r} = {}^{40}C_{r^2}$$

$$\Rightarrow r^2 = 3r \text{ or } r^2 = 40 - 3r$$

$$\Rightarrow r = 0, 3 \text{ or } -8, 5$$

3 and 5 are the values as the given equation is not defined by  $r = 0$  and  $r = -8$ . Hence, the number of values of  $r$  is 2.

14. (b) The thousandth place can be filled up in 9 ways with any one of the digits 1, 2, 3, ..., 9. After that the other three places can be filled up in  ${}^9P_3$

ways, with any one of the remaining 9 digits including zero. Hence, the number of four digit numbers with distinct digits =  $9 \times {}^9P_3$ .

15. (b) We have,  ${}^nP_r = {}^nP_{r+1}$

$$\Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!} \Rightarrow \frac{1}{(n-r)} = 1$$

$$\text{or } n-r=1 \quad \dots(1)$$

$$\text{Also, } {}^nC_r = {}^nC_{r-1}$$

$$\Rightarrow r+r-1=n$$

$$\Rightarrow 2r-n=1 \quad \dots(2)$$

Solving (1) and (2), we get  $r=2$  and  $n=3$

16. (c)  ${}^nP_r = 720 {}^nC_r$

$$\text{or } \frac{n!}{(n-r)!} = \frac{720(n!)}{(n-r)!r!}$$

$$\Rightarrow r! = 720 = 1 \times 2 \times 3 \times 4 \times 5 \times 6!$$

$$\text{or } r=6$$

17. (b) Since each bulb has two choices, either switched on or off, therefore required number =  $2^{10} - 1 = 1023$ .

18. (b) Here we have to divide 52 cards into 4 sets, three of them having 17 cards each and the fourth one having just one card. First we divide 52 cards into two groups of 1 card and 51 cards. this can

$$\text{be done in } \frac{52!}{1!51!} \text{ ways.}$$

Now every group of 51 cards can be divided into 3

$$\text{groups of 17 each in } \frac{51!}{(17!)^3 3!}.$$

Hence the required number of ways

$$= \frac{52!}{1!51!} \cdot \frac{51!}{(17!)^3 3!} = \frac{52!}{(17!)^3 3!}$$

19. (b) When 0 is the repeated digit like

100, 200, ..., 9 in number

When 0 occurs only once like

110, 220, ..., 9 in number

When 0 does not occur like

112, 211, ...,  $2 \times (8 \times 9) = 144$  in number.



