CH 5 CO-ORDINATE GEOMETRY

ANSWERS AND EXPLANATIONS

1. (c) Here $x_1 = 4$, $x_2 = -2$, $y_1 = -1$, $y_2 = 4$

and $m_1 = 3$ and $m_2 = 5$

$$\therefore x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$= \frac{3(-2)+5(4)}{3+5} = \frac{7}{4}$$

and $y = \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

$$= \frac{3(4)+5(-1)}{3+5} = \frac{7}{8}$$

$\therefore$ The required point is $\left(\frac{7}{4}, \frac{7}{8}\right)$

2. (a) Let the third vertex be (x, y)

$\therefore$ The centroid of the triangle is given (6, 1).

$$\Rightarrow \frac{x_1 + x_2 + x_3}{3} = 6 \Rightarrow \frac{3+11+x}{3} = 6 \Rightarrow 14 + x = 18$$

$\Rightarrow x = 4$

and $\dfrac{y_1 + y_2 + y_3}{3} = 1 \Rightarrow \dfrac{2+4+y}{3} = 1 \Rightarrow 6 + y = 3$

$y = -3$

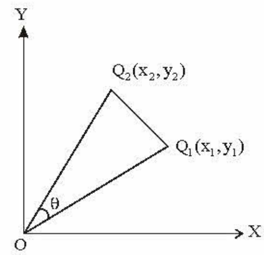
$\therefore$ Third vertex is (4, –3)

3. (b) Ratio $= -\left(\dfrac{-1+1-4}{5+7-4}\right) = \dfrac{1}{2}$

4. (c) Let fourth vertex be (x, y), then $\dfrac{x+8}{2} = \dfrac{2+5}{2}$

and $\dfrac{y+4}{2} = \dfrac{-2+7}{2} \Rightarrow x = -1, y = 1$

5. (c) From triangle OQ_1Q_2, by applying cosine formula.



$$Q_1Q_2^2 = OQ_1^2 + OQ_2^2 - 2OQ_1.OQ_2 \cos Q_1OQ_2$$

or

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = x_1^2 + y_1^2 + x_2^2 + y_2^2 - 2OQ_1.OQ_2 \cos\theta$$

or

$$x_1x_2 + y_1y_2 = OQ_1.OQ_2 \cos Q_1OQ_2$$

6. (b) Slope of AB $= \dfrac{a+c-b-c}{b-a} = \dfrac{a-b}{b-a} = -1$

Slope of BC $= \dfrac{a+b-a-c}{c-b} = \dfrac{b-c}{c-b} = -1$

Hence, collinear.

7. (c) $\dfrac{2\times 5+1\,(a)}{2+1} = 4 \Rightarrow a = 2$

and $\dfrac{2\times 7+1\,(b)}{2+1} = 6 \Rightarrow b = 4$

8. (d) We have the mid-point of diagonal = (1, – 1) which should be the mid point of the other two points as well and which is not satisfied by any given alternative.

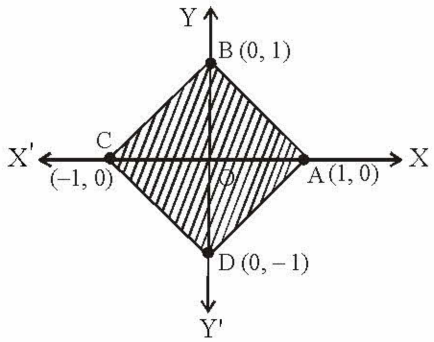
9. (b) $x = \frac{2+5+3}{3} = \frac{10}{3}$

and $y = \frac{1+2+4}{3} = \frac{7}{3}$

10. (d) $\frac{1}{2}[4-(2+16)+3(-16-4)+3(4+2)]$

$= \frac{1}{2}[56 - 60 + 18] = 7$

11. (c) The shaded region in the following graph satisfies given by the inequality.



Area of the shaded region = 4 × (Area of ΔAOB)

$= 4 \times \frac{1}{2} \times 1 \times 1 = 4 \times \frac{1}{2} = 2$ sq. units

12. (d) Let the equation of a line passing through the point of intersection of $3x - y - 1 = 0$ and $x - 3y + 5 = 0$ is

$(3x - y - 1) + k(x - 3y + 5) = 0$,

where k is a constant

Now, the line passes through (1, 5)

$\Rightarrow (3 - 5 - 1) + k(1 - 15 + 5) = 0$

or $k = \frac{-3}{9} = \frac{-1}{3}$

The required equation is

$(3x - y - 1) - \frac{1}{3}(x - 3y + 5) = 0$

or $9x - 3y - 3 - x + 3y - 5 = 0$

or $8x = 8$

or $x = 1$

13. (b) $y = x^3 + kx$

Slope at x = 2

$= \left(\frac{dy}{dx}\right)_{x=2} = [3x^2 + k]_{x=2}$

$= 3.2^2 + k = 12 + k$

Area of the curve $z = a^2 + a$ between a = 0 and a = 3 is given by $\int_0^3 (a^2 + a)da$

$= \left[\frac{a^3}{3} + \frac{a^2}{2}\right]_0^3 = \left[\frac{3^3}{3} + \frac{3^2}{2}\right] - 0$

$= 9 + \frac{9}{2} = \frac{27}{2}$

or $k = \frac{27}{2} - 12 = \frac{27-24}{2} = \frac{3}{2} = 1.5$

14. (d) Let the three points be P $\left(0, \frac{8}{3}\right)$, Q (1, 3) and R (82, 30).

Now, $PQ = \sqrt{(1-0)^2 + \left(3 - \frac{8}{3}\right)^2} = \frac{\sqrt{10}}{3}$,

$QR = \sqrt{(82-1)^2 + (30-3)^2}$

$= \sqrt{6561 + 729} = \sqrt{7290} = 27\sqrt{10}$,

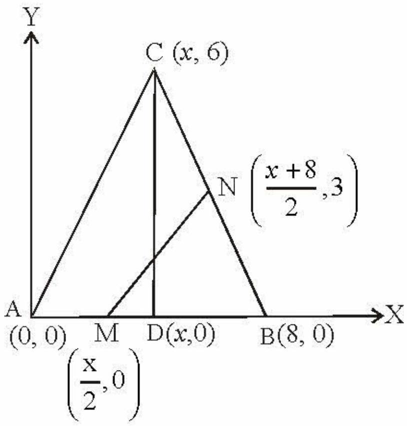
$RP = \sqrt{(82-0)^2 + \left(30 - \frac{8}{3}\right)^2}$

$= \sqrt{(82)^2 + \frac{(82)^2}{9}} = \frac{82}{3}\sqrt{10}$

Now, $PQ + QR = \frac{\sqrt{10}}{3} + 27\sqrt{10} = \frac{82\sqrt{10}}{3}$

Since PQ + QR = PR therefore, points P, Q, and R are collinear i.e. they form a straight line.

15. (d)



Let AD = x.

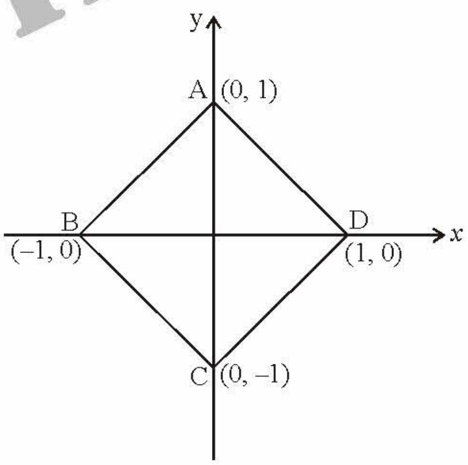
Co-ordinates of all the points are as shown in the figure above.

Now, required distance = MN

$$= \sqrt{\left(\frac{x}{2} - \frac{x+8}{2}\right)^2 + (0-3)^2}$$

$$= \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5\text{cm}$$

16. (b)



The slope of the equation y = – x + 1 is –1.

Hence, equation of line BC, passing through (–1, 0) and parallel to x + y = 1 is

$$(y - 0) = -1\ (x + 1)$$

$$y = -x - 1$$

$$x + y = -1$$

Equation of $AD \equiv x + y = 1$

Equation of $BC \equiv x + y = -1$

17. (c) The four equations are :

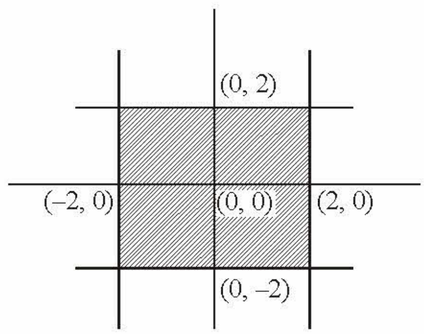
$x + y + x - y = 4$ or $x = 2$ ...(i)

$x + y - (x - y) = 4$ or $y = 2$ ...(ii)

$-(x + y) + x - y = 4$ or $y = -2$ ...(iii)

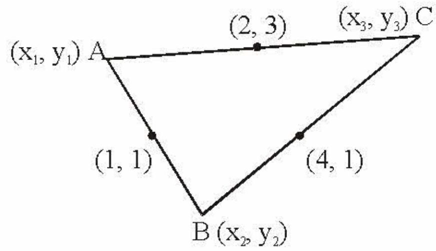
$-(x + y) - (x - y) = 4$ or $x = -2$ ...(iv)

The area bounded by the given curve is shown below



Hence, area = 4 × 4 = 16 sq. units.

18. (a)



Let the coordinates of the vertices be

$A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Then, we have

$x_1 + x_2 = 2,\ x_2 + x_3 = 8,\ x_3 + x_1 = 4$

and $y_1 + y_2 = 2,\ y_2 + y_3 = 2,\ y_3 + y_1 = 6$

From the above equations, we have

$x_1 + x_2 + x_3 = 7$

and $y_1 + y_2 + y_3 = 5$

Solving together, we have

$x_1 = -1,\ x_2 = 3,\ x_3 = 5$

and $y_1 = 3,\ y_2 = -1,\ y_3 = 3$

Therefore the coordinates of the vertices are

(–1, 3), (3, –1) and (5, 3).

Hence, the centroid is

$$\left(\frac{-1+3+5}{3}, \frac{3-1+3}{3}\right) \text{ i.e., } \left(\frac{7}{3}, \frac{5}{3}\right).$$

19. **(c)**

$$A \equiv \left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right),$$

Area of Δ ABC = 2 units

$$\Rightarrow \frac{1}{2}\left[\frac{3k-5}{k+1}(5+2)+1\left(-2-\frac{5k+1}{k+1}\right)+7\left(\frac{5k+1}{k+1}-5\right)\right]$$

$$= \pm 2$$

$\Rightarrow 14k - 66 = \pm 4(k+1) \Rightarrow k = 7$ or $31/9$