CH 4 PROBABILITY

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (b) Reqd probability = $\frac{^5C_2}{^7C_2} = \frac{5\times4}{7\times6} = \frac{10}{21}$

2. (e) If the drawn ball is neither red nor green, then it must be blue, which can be picked in $^7C_1$ = 7 ways. One ball can be picked from the total (8 + 7 + 6 = 21) in $^{21}C_1$ = 21 ways.

$\therefore$ Reqd probability $= \frac{7}{21} = \frac{1}{3}$

3. (b) $n(S) = {}^{12}c_3 = \frac{12\times11\times10}{3\times2} = 2\times11\times10 = 220$

No. of selection of 3 oranges out of the total 12 oranges =

${}^{12}c_3 = 2\times11\times10 = 220$.

No. of selection of 3 bad oranges out of the total 4 bad oranges = $^4c_3$ = 4

$\therefore$ $n(E)$ = no. of desired selection of oranges

= 220 – 4 = 216

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{216}{220} = \frac{54}{55}$

4. (b) Total no. of ways of drawing 3 marbles

$= {}^{12}C_3 = \frac{12\times11\times10}{1\times2\times3} = 220$

Total no. of ways of drawing marbles, which are of same colour

= $^5C_3$ + $^4C_3$ + $^3C_3$

= 10 + 4 + 1 = 15

$\therefore$ Probability of same colour = $\frac{15}{220} = \frac{3}{44}$

$\therefore$ Probalitity of not same colour = $1 - \frac{3}{44} = \frac{41}{42}$

5. (a) When two are thrown then there are 6 × 6 exhaustive cases \ n = 36. Let A denote the event "total score of 7" when 2 dice are thrown then A = [(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)].

Thus there are 6 favourable cases.

$\therefore$ m = 6 By definition $P(A) = \frac{m}{n}$

$\therefore P(A) = \frac{6}{36} = \frac{1}{6}$.

6. (c) There are 5 + 7 = 12 balls in the bag and out of these two balls can be drawn in $^{12}C_2$ ways. There are 5 green balls, therefore, one green ball can be drawn in $^5C_1$ ways; similarly, one red ball can be drawn in $^7C_1$ ways so that the number of ways in which we can draw one green ball and the other red is $^5C_1 \times {}^7C_1$.

Hence, P (one green and the other red)

$= \frac{^5C_1 \times {}^7C_1}{^{12}C_2} = \frac{5}{1}\times\frac{7}{1}\times\frac{1.2}{12.11} = \frac{35}{66}$.

7. (b) There are 7 + 5 = 12 balls in the bag and the number of ways in which 4 balls can be drawn is $^{12}C_4$ and the number of ways of drawing 4 black balls (out of seven) is $^7C_4$.

Hence, P (4 black balls)

$= \frac{^7C_4}{^{12}C_4} = \frac{7.6.5.4}{1.2.3.4}\times\frac{1.2.3.4}{12.11.10.9} = \frac{7}{99}$

Thus the odds against the event 'all black balls' are

$$(1-\frac{7}{99}):\frac{7}{99} \text{ : i.e., } \frac{92}{99}:\frac{7}{99} \text{ or } 92:7.$$

8. (b) The word 'SOCIETY' contains seven distinct letters and they can be arranged at random in a row in $^7P_7$ ways, i.e. in 7! = 5040 ways.

Let us now consider those arrangements in which all the three vowels come together. So in this case we have to arrange four letters. S,C,T,Y and a pack of three vowels in a row which can be done in $^5P_5$ i.e. 5! = 120 ways.

Also, the three vowels in their pack can be arranged in $^3P_3$ i.e. 3! = 6 ways.

Hence, the number of arrangements in which the three vowels come together is 120 × 6 = 720

∴ The probability that the vowels come together

$$=\frac{720}{5040}=\frac{1}{7}$$

9. (c) Let W stand for the winning of a game and L for losing it. Then there are 4 mutually exclusive possibilities

(i) W, W, W (ii) W, W, L, W

(iii) W, L, W, W (iv) L, W, W, W.

[Note that case (i) includes both the cases whether he losses or wins the fourth game.]

By the given conditions of the question, the probabilities for (i), (ii), (iii) and (iv) respectively are

$$\frac{2}{3}.\frac{2}{3}.\frac{2}{3};\ \frac{2}{3}.\frac{2}{3}.\frac{1}{3}.\frac{1}{3};\ \frac{2}{3}.\frac{1}{3}.\frac{1}{3}.\frac{2}{3} \text{ and } \frac{1}{3}.\frac{1}{3}.\frac{2}{3}.\frac{2}{3}.$$

Hence the required probability

$$=\frac{8}{27}+\frac{4}{81}+\frac{4}{81}+\frac{4}{81}=\frac{36}{81}=\frac{4}{9}.$$

[∵ The probability of winning the game if previous game was also won is $\frac{2}{1+2}=\frac{2}{3}$ and the probability of winning the game if previous game was a loss is $\frac{1}{1+2}=\frac{1}{3}$].

10. (b) The number of ways of choosing three numbers out of N is $^NC_3$. If these numbers are a_1, a_2 and a_3, they must satisfy exactly one of the following inequalities for a successful outcome.

$$a_1<a_2<a_3 \quad a_1<a_3<a_2, \quad a_2<a_1<a_3,$$
$$a_2<a_3<a_1, \quad a_3<a_1<a_2, \quad a_3<a_2<a_1.$$

Thus the number of ways of arranging the three numbers in a given order is ($^NC_3$) (6), and there are 3 ways in which the first number is less than the second. Now if A denotes the event : the first number is less than the second number, and B the event : the third number lies between the first and the second, we need to find P(B|A). Since

$$P(B\cap A)=\frac{^NC_3}{(^NC_3)(6)}=\frac{1}{6} \text{ and } P(A)=\frac{(^NC_3)(3)}{(^NC_3)(6)}=\frac{1}{2},$$

We get $P(B\mid A)=\frac{P(B\cap A)}{P(A)}=\frac{1/6}{1/2}=\frac{1}{3}.$

11. (b) Suppose E_1, E_2 and E_3 are the events of winning the race by the horses A, B and C respectively

$$\therefore P(E_1)=\frac{1}{1+3}=\frac{1}{4}, \quad P(E_2)=\frac{1}{1+4}=\frac{1}{5}$$

$$P(E_3)=\frac{1}{1+5}=\frac{1}{6}$$

∴ Probability of winning the race by one of the horses A, B and C

$$=P(E_1 \text{ or } E_2 \text{ or } E_3)=P(E_1)+P(E_2)+P(E_3)$$

$$=\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=\frac{37}{60}$$

12. (d) Probability that only husband is selected

$$=P(H)P(\overline{W})=\frac{1}{7}\left(1-\frac{1}{5}\right)=\frac{1}{7}\times\frac{4}{5}=\frac{4}{35}$$

Probability that only wife is selected