

Answer Key

1. (a) $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots \infty}}}$

On squaring,

$$x^2 = 6 + \sqrt{6 + \sqrt{6 + \dots \infty}}$$

$$x^2 = 6 + x$$

$$x^2 - x - 6 = 0$$

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x - 3) + 2(x - 3) = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3 \text{ because } x \neq -2$$

2. (a) $x + \frac{1}{x} = 3$

On squaring,

$$\left(x + \frac{1}{x}\right)^2 = 9$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 = 7$$

Again, $\left(x + \frac{1}{x}\right)^3 = 27$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 27 - 3 \times 3 = 18$$

$$\therefore \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) = 7 \times 18$$

$$\Rightarrow x^5 + \frac{1}{x^5} + \left(x + \frac{1}{x}\right) = 126$$

$$\Rightarrow x^5 + \frac{1}{x^5} = 126 - 3 = 123$$

3. (b) If $a + b + c = 0$
then $a^3 + b^3 + c^3 - 3abc = 0$

4. (a) $a^3 + b^3 + c^3 - 3abc$

$$= (a + b + c)$$

$$(a^2 + b^2 + c^2 - ab - bc - ac)$$

$$= \frac{1}{2}(a + b + c)(2a^2 + 2b^2 +$$

$$2c^2 - 2ab - 2bc - 2ac)$$

$$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

$$\frac{a^3 + b^3 + c^3 - 3abc}{a + b + c}$$

$$= \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

$$= \frac{1}{2}(9 + 25 + 1)$$

$$= \frac{35}{2} = 17.5$$

5. (d) $1^2 + 2^2 + 3^2 + \dots + n^2$

$$\therefore = \frac{n(n+1)(2n+1)}{6}$$

Required average

$$= \frac{(n+1)(2n+1)}{6}$$

$$= \frac{(10+1)(2 \times 10 + 1)}{6}$$

$$= \frac{11 \times 21}{6} = \frac{77}{2} = 38.5$$

6. (c) $2(a^2 + b^2)$

$$= (a + b)^2 + (a - b)^2$$

$$= (6)^2 + (2)^2$$



$$= 36 + 4 = 40$$

$$7. (b) \sqrt{2x} \times \frac{5}{100} = 0.01$$

$$\Rightarrow \sqrt{2x} \times 5 = 0.01 \times 100 = 1$$

On squaring,

$$2x \times 25 = 1$$

$$\Rightarrow x = \frac{1}{50} = 0.02$$

$$8. (b) \sqrt{\frac{x-a}{x-b}} - \sqrt{\frac{x-b}{x-a}} = \frac{b}{x} - \frac{a}{x}$$

$$\Rightarrow \sqrt{\frac{(x-a)(x-a)}{(x-b)(x-a)}} - \sqrt{\frac{(x-b)(x-b)}{(x-a)(x-b)}}$$

$$= \frac{b-a}{x}$$

$$\Rightarrow \frac{x-a}{\sqrt{(x-b)(x-a)}} - \frac{x-b}{\sqrt{(x-a)(x-b)}}$$

$$= \frac{b-a}{x}$$

$$\Rightarrow \frac{x-a-x+b}{\sqrt{(x-b)(x-a)}} = \frac{b-a}{x}$$

$$\Rightarrow \frac{b-a}{\sqrt{(x-b)(x-a)}} = \frac{b-a}{x}$$

$$\Rightarrow x = \sqrt{(x-b)(x-a)}$$

On squaring,

$$x^2 = (x-b)(x-a)$$

$$\Rightarrow x^2 = x^2 - ax - bx + ab$$

$$\Rightarrow ax + bx = ab$$

$$\Rightarrow x(a+b) = ab \Rightarrow x = \frac{ab}{a+b}$$

$$9. (b) x = \frac{2\sqrt{24}}{\sqrt{3} + \sqrt{2}}$$

$$\Rightarrow x = \frac{2\sqrt{3} \times 8}{\sqrt{3} + \sqrt{2}} = \frac{2\sqrt{3} \times \sqrt{8}}{\sqrt{3} + \sqrt{2}}$$

$$\Rightarrow \frac{x}{\sqrt{8}} = \frac{2\sqrt{3}}{\sqrt{3} + \sqrt{2}}$$

$$\Rightarrow \frac{x + \sqrt{8}}{x - \sqrt{8}} = \frac{2\sqrt{3} + \sqrt{3} + \sqrt{2}}{2\sqrt{3} - \sqrt{3} - \sqrt{2}}$$

(By componendo and dividendo)

$$= \frac{3\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

Again,

$$\frac{x}{\sqrt{12}} = \frac{2\sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$\Rightarrow \frac{x + \sqrt{12}}{x - \sqrt{12}}$$

$$= \frac{2\sqrt{2} + \sqrt{3} + \sqrt{2}}{2\sqrt{2} - \sqrt{3} - \sqrt{2}} = \frac{\sqrt{3} + 3\sqrt{2}}{\sqrt{2} - \sqrt{3}}$$

$$\therefore \frac{x + \sqrt{8}}{x - \sqrt{8}} + \frac{x + \sqrt{12}}{x - \sqrt{12}}$$

$$\frac{3\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} + \frac{\sqrt{3} + 3\sqrt{2}}{\sqrt{2} - \sqrt{3}}$$

$$\frac{3\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} + \frac{\sqrt{3} + 3\sqrt{2}}{\sqrt{2} - \sqrt{3}}$$

$$= \frac{3\sqrt{3} + \sqrt{2} - \sqrt{3} - 3\sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$\frac{2\sqrt{3} - 2\sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$= \frac{2(\sqrt{3} - \sqrt{2})}{\sqrt{3} - \sqrt{2}} = 2$$

$$10. (a) \text{ Expression}$$

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$$



$$\begin{aligned}
 &= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{4 + 3 + 2.2\sqrt{3}}}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{(2 + \sqrt{3})^2}}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{3 + 8(2 + \sqrt{3})}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{19 + 8\sqrt{3}}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{16 + 3 + 2.4\sqrt{3}}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{(4 + \sqrt{3})^2}} \\
 &= \sqrt{-\sqrt{3} + 4 + \sqrt{3}} = \sqrt{4} = 2
 \end{aligned}$$

11. (c) $x = \frac{1}{2 + \sqrt{3}} = \frac{2 - \sqrt{3}}{(2 + \sqrt{3})(2 - \sqrt{3})}$

$$= \frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$$

$$x + 1 = 3 - \sqrt{3}$$

Similarly, $y + 1 = 3 + \sqrt{3}$

$$\therefore \frac{1}{x + 1} + \frac{1}{y + 1}$$

$$\frac{1}{3 - \sqrt{3}} + \frac{1}{3 + \sqrt{3}}$$

$$= \frac{3 + \sqrt{3} + 3 - \sqrt{3}}{(3 - \sqrt{3})(3 + \sqrt{3})}$$

$$= \frac{6}{9 - 3} = 1$$

12. (c) $x = 2 - \frac{1}{2^3} + \frac{2}{2^3}$

$$\Rightarrow x - 2 = \frac{2}{2^3} - \frac{1}{2^3}$$

On Cubing

$$x^3 - 3x^2 \times 2 + 3x \times 4 - 8$$

$$= \left(2\frac{2}{3}\right)^3 - \left(\frac{1}{2^3}\right)^3$$

$$= 3 \cdot 2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}} \left(2^{\frac{2}{3}} - 2^{\frac{1}{3}}\right)$$

$$\Rightarrow x^3 - 6x^2 + 12x - 8 = 4 - 2 - 6(x - 2)$$

$$\Rightarrow x^3 - 6x^2 + 12x - 8$$

$$= 2 - 6x + 12$$

$$\Rightarrow x^3 - 6x^2 + 18x + 18$$

$$= 2 + 12 + 8 + 18 = 40$$

13. (d) $a^3 + b^3 + c^3 - 3abc = 0$

$$\text{If } a + b + c = 0$$

$$\therefore a^3 - b^3 - c^3 - 3abc = 0$$

$$\Rightarrow a - b - c = 0$$

$$\Rightarrow a = b + c$$

14. (c) $P(x) = ax^3 + 3x^2 - 8x + b$

$$\therefore P(-2) = -8a = 12 + 16 + b = 0$$

$$\Rightarrow -8a + b + 28 = 0 \quad \dots\dots(i)$$

$$\Rightarrow P(2) = 8a + 12 - 16 + b = 2$$

$$\Rightarrow 8a + b - 4 = 0 \quad \dots\dots(ii)$$

By equation (i) + (iii)

$$2b + 24 = 0$$

$$\Rightarrow b = -\frac{24}{2} = -12$$

From equation (i),

$$-8a - 12 + 28 = 0$$

$$\Rightarrow -8a = -16$$

$$\Rightarrow a = 2$$

15. (b) $2x + y = 5 \quad \dots\dots(i)$

$$x + 2y = 4 \quad \dots\dots(ii)$$

By equation (i) $\times 2$ - equation (ii), we have

$$4x + 2y = 10$$



$$x + 2y = 4$$

$$\underline{\quad\quad\quad}$$

$$3x = 6$$

$$\Rightarrow x = 2$$

From equation (i),

$$2 \times 2 + y = 5$$

$$\Rightarrow y = 5 - 4 = 1$$

\(\therefore\) Point of intersection = (2, 1)

16. (a) $2^x \cdot 2^y = 8$

$$\Rightarrow 2^{x+y} = 2^3$$

$$\Rightarrow x + y = 3$$

$$9^x \cdot 3^y = 3^4$$

$$\Rightarrow 3^{2x} \cdot 3^y = 3^4$$

$$\Rightarrow 2x + y = 4$$

By equation (i),

$$x = 1$$

From equation (i),

$$1 + y = 3$$

$$\Rightarrow y = 2$$

17. (b) $x = 3 + 2\sqrt{2}$

$$\therefore \frac{1}{x} = \frac{1}{3 + 2\sqrt{2}}$$

$$= \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$$

$$= \frac{3 - 2\sqrt{2}}{9 - 8} = 3 - 2\sqrt{2}$$

$$\therefore \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 = x + \frac{1}{x} - 2$$

$$= 3 + 2\sqrt{2} + 3 - 2\sqrt{2} - 2 = 4$$

$$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = 2$$

18. (a) $25a^2 + 40ab + 16b^2$

$$= (5a + 4b)^2$$

$$= (5 \times 23 - 29 \times 4)^2$$

$$= (115 - 116)^2 = 1$$

19. (a) $\frac{a}{3} = \frac{b}{2} \Rightarrow \frac{a}{b} = \frac{3}{2}$

$$\therefore \frac{2a + 3b}{3a - 2b} = \frac{2 \times \frac{a}{b} + 3}{3 \times \frac{a}{b} - 2}$$

$$= \frac{2 \times \frac{3}{2} + 3}{3 \times \frac{3}{2} - 2} = \frac{6}{\frac{9-4}{2}} = \frac{12}{5}$$

20. (b) Sum of x numbers = xy

Sum of y numbers = xy

\(\therefore\) Required average

$$= \frac{xy + xy}{x + y} = \frac{2xy}{x + y}$$

21. (a) $x + \frac{1}{4x} = \frac{3}{2}$

$$\Rightarrow 2x + \frac{1}{2x} = 3$$

Cubing both sides,

$$8x^3 + \frac{1}{8x^3} + 3 \times 2x \times \frac{1}{2x}$$

$$\left(2x + \frac{1}{2x} \right) = 27$$

$$\Rightarrow 8x^3 + \frac{1}{8x^3} + 3 \times 3 = 27$$

$$\Rightarrow 8x^3 + \frac{1}{8x^3} = 27 - 9 = 18$$

22. (d) $x = \frac{4ab}{a+b} \Rightarrow \frac{x}{2a} = \frac{2b}{a+b}$

By componendo and dividendo,



