CH 2 PIPES AND CISTERNS

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (c) Part of the tank filled in one hour

$= \frac{1}{8} - \frac{1}{16} = \frac{1}{16}$

Hence, the tank will be filled in 16 hours.

2. (d) Let the exhaust tap empties the tank in x minutes.

Then, $\frac{1}{12} + \frac{1}{15} - \frac{1}{x} = \frac{1}{20}$

or $\frac{1}{x} = \frac{1}{12} + \frac{1}{15} - \frac{1}{20}$

or $\frac{1}{x} = \frac{5+4-3}{60} = \frac{6}{60} = \frac{1}{10}$ or x = 10 min

3. (c) Let the leak empties the tank in x hours.

Now, $\frac{1}{5} - \frac{1}{x} = \frac{1}{6}$

or $\frac{1}{x} = \frac{1}{5} - \frac{1}{6} = \frac{1}{30}$

or x = 30 hrs.

4. (c) Let pipe A fills the cistern in x minutes.

Therefore, pipe B will fill the cistern in (x + 5) minutes.

Now, $\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$

$\Rightarrow$ x = 10

Thus, the pipes A and B can fill the cistern respectively in 10 minutes and 15 minutes,

5. (a) Portion of the tank filled by all the pipes together in 1 hour $= \frac{1}{10} + \frac{1}{12} - \frac{1}{20}$

$= \frac{6+5-3}{60} = \frac{8}{60} = \frac{2}{15}$

Hence, the tank will be filled in $\frac{15}{2}$ hours

or $7\frac{1}{2}$ hours.

6. (a) Part of the capacity of the cistern emptied by the leak in one hour

$= \left(\frac{1}{6} - \frac{1}{7}\right) = \frac{1}{42}$ of the cistern.

The whole cistern will be emptied in 42 hours.

7. (b) Work of both tap for 1 hour

$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Hence, both tap will fill the cistern in 6 hours.

8. (c) Part filled by (A + B + C) in 1 hour

$= \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{30}\right) = \frac{1}{3}$.

$\therefore$ All the three pipes together will fill the tank in 3 hours.

9. (c) Work done by the waste pipe in 1 minute

$= \frac{1}{15} - \left(\frac{1}{20} + \frac{1}{24}\right) = \left(\frac{1}{15} - \frac{11}{120}\right) = -\frac{1}{40}$.

[–ve sign means emptying]

$\therefore$ Volume of $\frac{1}{40}$ part = 3 gallons.

Volume of whole = (3 × 40) gallons = 120 gallons.

10. (d) Capacity of the tank = (12 × 13.5) litres = 162 litres.

Capacity of each bucket = 9 litres.

Number of buckets needed

$= \left(\frac{162}{9}\right) = 18.$

11. (d) Tank filled in 1 minute

$= \frac{1}{25} + \frac{1}{40} - \frac{1}{30}$ part

$= \frac{24+15-20}{600} = \frac{19}{600}$ part

$\therefore$ tank will be filled complete in minutes

$= \frac{600}{19} = 31\frac{11}{19}$

12. (a) Here ratio of efficiencies of pipes A, B and C are as follows:

C		B		A
2		1		
		2		1
4	:	2	:	1

Suppose the efficiencies of pipes C, B and A are 4K, 2K and K.

Since, the tank is filled in 5 hours by the three pipes having combined efficiency equal to 7K, the time required to fill the tank by A alone

$= \frac{7K \times 5}{K} = 35$ hours

13. (a)

$\because$ Pipe *A* in 1 minute fills 1/10 part and Pipe *B* in 1 min. empties $\frac{1}{6}$ part

$\therefore$ Pipe A + B in 1 min $= \frac{1}{10} - \frac{1}{6} = \frac{-1}{15}$

$\because$ $\frac{1}{15}$ part gets emptied in 1 min

$\therefore$ $\frac{2}{5}$ part is emptied in $15 \times \frac{2}{5}$ min = 6 min

EXERCISE 2

1. (d) Capacity of water throwing pump

$= \frac{12}{60} \times 5.5 = 1.1$ tonnes per 5.5 minutes

Capacity of the leak to admit water

= 2.25 tonnes per 5.5 minutes

In 5.5 minutes, net water accumulated by the leak

= (2.25 – 1.1) = 1.15 tonnes

Thus, to admit 92 tonnes of water, it will take

$\frac{5.5}{1.15} \times 92$

= 440 min $= \frac{440}{60}$ hrs

Speed required for the ship to sail through safely

$= \frac{77 \times 60}{440} = 10.5 \text{km/h}$

2. (c) If both the pumps are opened together, then the tank will be emptied because the working efficiency of pump empting is more than that of the pump filling it. Thus in 1 min net work done

$= \left(\frac{1}{8} - \frac{1}{16}\right) = \frac{1}{16}$ parts

or the tank will be emptied in 16 min

$\Rightarrow$ $\frac{1}{2}$ tank will be emptied in 8 min.

3. (c) Proportion of the volume of the tank filled by both the pipes in 4 min

$= 4\left(\frac{1}{15} + \frac{1}{10}\right) = \frac{2}{3}$rd of the tank.

Volume of the tank filled by all the pipes working together

$= \frac{1}{15} + \frac{1}{10} - \frac{1}{5} = \frac{-1}{30}$

i.e. $\frac{1}{30}$ tank is emptied in 1 min.

4. (d) A + B fill in 6 hrs.

B + C fill in 10 hrs.

A + C fill in $7\frac{1}{2} = \frac{15}{2}$ hrs

$\therefore$ 2 (A + B + C) fill in

$$\frac{6\times10\times\frac{15}{2}}{6\times10+6\times\frac{15}{2}+10\times\frac{15}{2}}$$

$$= \frac{6\times5\times15}{180} = \frac{5}{2}$$

$\therefore$ A + B + C filled the tank in 5 hrs.

Now, A[= (A + B + C) – (B + C)] fill in

$$\frac{10\times5}{10-5} = 10\text{hrs.}$$

Similarly, B fill in

$$\frac{\frac{15}{2}\times5}{\frac{15}{2}-5} = 15 \text{ hrs and}$$

C fill in $\frac{5\times6}{6-5} = 30$ hrs.

5. (c) Net part filled in 1 hour

$$= \left(\frac{1}{5}+\frac{1}{6}-\frac{1}{12}\right)=\frac{17}{60}.$$

$\therefore$ The tank will be full in

$\frac{60}{17}$ hrs i.e., $3\frac{9}{17}$ hrs.

6. (d) Part filled by first tap in one min $= \frac{1}{12}$th

Part filled by second tap in one min $= \frac{1}{18}$th

Now, $2\left[\frac{1}{12}+\frac{1}{18}\right]$ + unfilled part = 1

$\Rightarrow$ unfilled part = $\frac{13}{18}$th

$\because$ $\frac{1}{18}$th part of tank is filled by second tap in 1min.

$\therefore$ $\frac{13}{18}$th part of tank is filled by second tap in 1 min.

$$= 18\times\frac{13}{18}\text{ min} = 13\text{ min.}$$

7. (b) In one min, (A + B) fill the cistern

$$= \frac{1}{10}+\frac{1}{15}=\frac{1}{6}\text{th}$$

In 3 min, (A + B) fill the cistern = $\frac{3}{6}=\frac{1}{2}$th

Remaining part $= 1-\frac{1}{2}=\frac{1}{2}$

$\because$ $\frac{1}{10}$th part filled by A in one min.

$\therefore$ $\frac{1}{2}$nd part filled by A in $10\times\frac{1}{2}=5\text{min}$.

$\therefore$ Total time = 3 + 5 = 8 min.

8. (c) 1 minute's work of each of the three pipes

$$= \frac{1}{20}+\frac{1}{30}-\frac{1}{15}=\frac{3+2-4}{60}=\frac{1}{60}$$

i.e., work of 3 pipes for 3 minutes $= \frac{1}{60}$

$\therefore$ Work of 3 pipes for 55 minutes each

$$= \frac{1}{60}\times55 = \frac{11}{12}$$

$\therefore$ Remaining part to be filled

$$= 1-\frac{11}{12}=\frac{1}{12}$$

Now, pipe A will fill $\frac{1}{20}$ of the cistern in next 1 minute.