

CH 4 PROBABILITY

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (b) Req'd probability = $\frac{{}^5C_2}{{}^7C_2} = \frac{5 \times 4}{7 \times 6} = \frac{10}{21}$

2. (e) If the drawn ball is neither red nor green, then it must be blue, which can be picked in ${}^7C_1 = 7$ ways. One ball can be picked from the total $(8 + 7 + 6 = 21)$ in ${}^{21}C_1 = 21$ ways.

\therefore Req'd probability = $\frac{7}{21} = \frac{1}{3}$

3. (b) $n(S) = {}^{12}C_3 = \frac{12 \times 11 \times 10}{3 \times 2} = 2 \times 11 \times 10 = 220$

No. of selection of 3 oranges out of the total 12 oranges =

${}^{12}C_3 = 2 \times 11 \times 10 = 220$.

No. of selection of 3 bad oranges out of the total 4 bad oranges = ${}^4C_3 = 4$

\therefore $n(E) =$ no. of desired selection of oranges

$= 220 - 4 = 216$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{216}{220} = \frac{54}{55}$

4. (b) Total no. of ways of drawing 3 marbles

$= {}^{12}C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$

Total no. of ways of drawing marbles, which are of same colour

$= {}^5C_3 + {}^4C_3 + {}^3C_3$

$= 10 + 4 + 1 = 15$

\therefore Probability of same colour = $\frac{15}{220} = \frac{3}{44}$

\therefore Probability of not same colour = $1 - \frac{3}{44} = \frac{41}{44}$

5. (a) When two are thrown then there are 6×6 exhaustive cases $\therefore n = 36$. Let A denote the event "total score of 7" when 2 dice are thrown then $A = [(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)]$.

Thus there are 6 favourable cases.

$\therefore m = 6$ By definition $P(A) = \frac{m}{n}$

$\therefore P(A) = \frac{6}{36} = \frac{1}{6}$.

6. (c) There are $5 + 7 = 12$ balls in the bag and out of these two balls can be drawn in ${}^{12}C_2$ ways. There are 5 green balls, therefore, one green ball can be drawn in 5C_1 ways; similarly, one red ball can be drawn in 7C_1 ways so that the number of ways in which we can draw one green ball and the other red is ${}^5C_1 \times {}^7C_1$.

Hence, P (one green and the other red)

$= \frac{{}^5C_1 \times {}^7C_1}{{}^{12}C_2} = \frac{5}{1} \times \frac{7}{1} \times \frac{1.2}{12.11} = \frac{35}{66}$.

7. (b) There are $7 + 5 = 12$ balls in the bag and the number of ways in which 4 balls can be drawn is ${}^{12}C_4$ and the number of ways of drawing 4 black balls (out of seven) is 7C_4 .

Hence, P (4 black balls)

$= \frac{{}^7C_4}{{}^{12}C_4} = \frac{7.6.5.4}{1.2.3.4} \times \frac{1.2.3.4}{12.11.10.9} = \frac{7}{99}$

Thus the odds against the event 'all black balls' are

$$(1 - \frac{7}{99}) : \frac{7}{99} \text{ i.e., } \frac{92}{99} : \frac{7}{99} \text{ or } 92 : 7.$$

8. (b) The word 'SOCIETY' contains seven distinct letters and they can be arranged at random in a row in 7P_7 ways, i.e. in $7! = 5040$ ways.

Let us now consider those arrangements in which all the three vowels come together. So in this case we have to arrange four letters. S,C,T,Y and a pack of three vowels in a row which can be done in 5P_5 i.e. $5! = 120$ ways.

Also, the three vowels in their pack can be arranged in 3P_3 i.e. $3! = 6$ ways.

Hence, the number of arrangements in which the three vowels come together is $120 \times 6 = 720$

\therefore The probability that the vowels come together

$$= \frac{720}{5040} = \frac{1}{7}$$

9. (c) Let W stand for the winning of a game and L for losing it. Then there are 4 mutually exclusive possibilities

- (i) W, W, W (ii) W, W, L, W
(iii) W, L, W, W (iv) L, W, W, W.

[Note that case (i) includes both the cases whether he losses or wins the fourth game.]

By the given conditions of the question, the probabilities for (i), (ii), (iii) and (iv) respectively are

$$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}, \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}, \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} \text{ and } \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}.$$

Hence the required probability

$$= \frac{8}{27} + \frac{4}{81} + \frac{4}{81} + \frac{4}{81} = \frac{36}{81} = \frac{4}{9}.$$

[\therefore The probability of winning the game if previous game was also won is $\frac{2}{1+2} = \frac{2}{3}$ and the probability of winning the game if previous game

was a loss is $\frac{1}{1+2} = \frac{1}{3}$].

10. (b) The number of ways of choosing three numbers out of N is NC_3 . If these numbers are a_1, a_2 and a_3 , they must satisfy exactly one of the following inequalities for a successful outcome.

$$a_1 < a_2 < a_3 \quad a_1 < a_3 < a_2, \quad a_2 < a_1 < a_3, \\ a_2 < a_3 < a_1, \quad a_3 < a_1 < a_2, \quad a_3 < a_2 < a_1.$$

Thus the number of ways of arranging the three numbers in a given order is $({}^NC_3)(6)$, and there are 3 ways in which the first number is less than the second. Now if A denotes the event : the first number is less than the second number, and B the event : the third number lies between the first and the second, we need to find $P(B|A)$. Since

$$P(B \cap A) = \frac{{}^NC_3}{({}^NC_3)(6)} = \frac{1}{6} \text{ and } P(A) = \frac{({}^NC_3)(3)}{({}^NC_3)(6)} = \frac{1}{2},$$

We get $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{1/6}{1/2} = \frac{1}{3}$.

11. (b) Suppose E_1, E_2 and E_3 are the events of winning the race by the horses A, B and C respectively

$$\therefore P(E_1) = \frac{1}{1+3} = \frac{1}{4}, \quad P(E_2) = \frac{1}{1+4} = \frac{1}{5}$$

$$P(E_3) = \frac{1}{1+5} = \frac{1}{6}$$

\therefore Probability of winning the race by one of the horses A, B and C

$$= P(E_1 \text{ or } E_2 \text{ or } E_3) = P(E_1) + P(E_2) + P(E_3) \\ = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{37}{60}$$

12. (d) Probability that only husband is selected

$$= P(H)P(\bar{W}) = \frac{1}{7} \left(1 - \frac{1}{5}\right) = \frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$$

Probability that only wife is selected

