Answer Key

1. (a) $x = \sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}$

On squaring,

$x^2 = 6+\sqrt{6+\sqrt{6+.....\infty}}$

$x^2 = 6+x$

$x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3)+2(x-3)=0$

$(x-3)(x+2)=0$

$x = 3$ because $x \neq -2$

2. (a) $x+\frac{1}{x}=3$

On squaring,

$\left(x+\frac{1}{x}\right)^2 = 9$

$\Rightarrow \quad x^2+\frac{1}{x^2}=9-2=7$

Again, $\left(x+\frac{1}{x}\right)^3 = 27$

$\Rightarrow \quad x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=27$

$\Rightarrow \quad x^3+\frac{1}{x^3}=27-3\times 3=18$

$\therefore \quad \left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)$

$= 7\times 18$

$\Rightarrow \quad x^5+\frac{1}{x^5}+\left(x+\frac{1}{x}\right)=126$

$\Rightarrow \quad x^5+\frac{1}{x^5}=126-3=123$

3. (b) If $a+b+c+0$

then $a^3+b^3+c^3-3abc=0$

4. (a) $a^3+b^3+c^3-3abc$

$=(a+b+c)$

$(a^2+b^2+c^2-ab-bc-ac)$

$=\frac{1}{2}(a+b+c)(2a^2+2b^2+$

$2c^2-2ab-2bc-2ac)$

$=\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$

$\frac{a^3+b^3+c^3-3abc}{a+b+c}$

$=\frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

$=\frac{1}{2}(9+25+1)$

$=\frac{35}{2}=17.5$

5. (d) $1^2+2^2+3^2+.....+n^2$

$\therefore \quad =\frac{n(n+1)(2n+1)}{6}$

Required average

$=\frac{(n+1(2n+1)}{6}$

$=\frac{(10+1)(2\times 10+1)}{6}$

$=\frac{11\times 21}{6}=\frac{77}{2}=38.5$

6. (c) $2(a^2+b^2)$

$=(a+b)^2+(a-b)^2$

$=(6)^2+(2)^2$

$= 36 + 4 = 40$

7. (b) $\sqrt{2x} \times \frac{5}{100} = 0.01$

$\Rightarrow \sqrt{2x} \times 5 = 0.01 \times 100 = 1$

On squaring,

$2x \times 25 = 1$

$\Rightarrow x = \frac{1}{50} = 0.02$

8. (b) $\sqrt{\frac{x-a}{x-b}} - \sqrt{\frac{x-b}{x-a}} = \frac{b}{x} - \frac{a}{x}$

$\Rightarrow \sqrt{\frac{(x-a)(x-a)}{(x-b)(x-a)}} - \sqrt{\frac{(x-b)(x-b)}{(x-a)(x-b)}}$

$= \frac{b-a}{x}$

$\Rightarrow \frac{x-a}{\sqrt{(x-b)(x-a)}} - \frac{x-b}{\sqrt{(x-a)(x-b)}}$

$= \frac{b-a}{x}$

$\Rightarrow \frac{x-a-x+b}{\sqrt{(x-b)(x-a)}} = \frac{b-a}{x}$

$\Rightarrow \frac{b-a}{\sqrt{(x-b)(x-a)}} = \frac{b-a}{x}$

$\Rightarrow x = \sqrt{(x-b)(x-a)}$

On squaring,

$x^2 = (x-b)(x-a)$

$\Rightarrow x^2 = x^2 - ax - bx + ab$

$\Rightarrow ax + bx = ab$

$\Rightarrow x(a+b) = ab \Rightarrow x = \frac{ab}{a+b}$

9. (b) $x = \frac{2\sqrt{24}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow x = \frac{2\sqrt{3} \times 8}{\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3} \times \sqrt{8}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{x}{\sqrt{8}} = \frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{x+\sqrt{8}}{x-\sqrt{8}} = \frac{2\sqrt{3}+\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{3}-\sqrt{2}}$

(By componendo and dividendo)

$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Again,

$\frac{x}{\sqrt{12}} = \frac{2\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{x+\sqrt{12}}{x-\sqrt{12}}$

$= \frac{2\sqrt{2}+\sqrt{3}+\sqrt{2}}{2\sqrt{2}-\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{3}}$

$\therefore \frac{x+\sqrt{8}}{x-\sqrt{8}} + \frac{x+\sqrt{12}}{x-\sqrt{12}}$

$\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{3}}$

$\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{2}}$

$= \frac{3\sqrt{3}+\sqrt{2}-\sqrt{3}-3\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$\frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{2(\sqrt{3}-\sqrt{2})}{\sqrt{3}-\sqrt{2}} = 2$

10. (a) Expression

$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7+4\sqrt{3}}}}$

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{4 + 3 + 2.2\sqrt{3}}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{(2+\sqrt{3})^2}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 8(2+\sqrt{3})}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{19 + 8\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{16 + 3 + 2.4\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{(4+\sqrt{3})^2}}$$

$$= \sqrt{-\sqrt{3} + 4 + \sqrt{3}} = \sqrt{4} = 2$$

11. (c) $x = \dfrac{1}{2+\sqrt{3}} = \dfrac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

$$= \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$$

$$x + 1 = 3 - \sqrt{3}$$

Similarly, $y + 1 = 3 + \sqrt{3}$

$$\therefore \quad \frac{1}{x+1} + \frac{1}{y+1}$$

$$\frac{1}{3-\sqrt{3}} + \frac{1}{3+\sqrt{3}}$$

$$= \frac{3+\sqrt{3}+3-\sqrt{3}}{(3-\sqrt{3})(3+\sqrt{3})}$$

$$= \frac{6}{9-3} = 1$$

12. (c) $x = 2 - 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

$$\Rightarrow \quad x - 2 = 2^{\frac{2}{3}} - 2^{\frac{1}{3}}$$

On Cubing

$$x^3 - 3x^2 \times 2 + 3x \times 4 - 8$$

$$= \left(2\frac{2}{3}\right)^3 - \left(2^{\frac{1}{3}}\right)^3$$

$$-3.2^{\frac{2}{3}}.2^{\frac{1}{3}}\left(2^{\frac{2}{3}} - 2^{\frac{1}{3}}\right)$$

$$\Rightarrow \quad x^3 - 6x^2 + 12x - 8 = 4 - 2 - 6(x - 2)$$

$$\Rightarrow \quad x^3 - 6x^2 + 12x - 8$$

$$= 2 - 6x + 12$$

$$\Rightarrow \quad x^3 - 6x^2 + 18x + 18$$

$$= 2 + 12 + 8 + 18 = 40$$

13. (d) $a^3 + b^3 + c^3 - 3abc = 0$

If $a + b + c = 0$

$$\therefore \quad a^3 - b^3 - c^3 - 3abc = 0$$

$$\Rightarrow \quad a - b - c = 0$$

$$\Rightarrow \quad a = b + c$$

14. (c) $P(x) = ax^3 + 3x^2 - 8x + b$

$$\therefore \quad P(-2) = -8a = 12 + 16 + b = 0$$

$$\Rightarrow \quad -8a + b + 28 = 0 \qquad \text{.....(i)}$$

$$\Rightarrow \quad P(2) = 8a + 12 - 16 + b = 2$$

$$\Rightarrow \quad 8a + b - 4 = 0 \qquad \text{.....(ii)}$$

By equation (i) + (iii)

$$2b + 24 = 0$$

$$\Rightarrow \quad b = -\frac{24}{2} = -12$$

From equation (i),

$$-8a - 12 + 28 = 0$$

$$\Rightarrow \quad -8a = -16$$

$$\Rightarrow \quad a = 2$$

15. (b) $2x + y = 5$ .....(i)

$x + 2y = 4$ .....(ii)

By equation (i) × 2 – equation (ii), we have

$$4x + 2y = 10$$

$x + 2y = 4$

$- \quad - \quad -$

$3x = 6$

$\Rightarrow x = 2$

From equation (i),

$2 \times 2 + y = 5$

$\Rightarrow y = 5 - 4 = 1$

$\therefore$ Point of intersection = (2, 1)

16. (a) $2^x.2^y = 8$

$\Rightarrow 2^{x+y} = 2^3$

$\Rightarrow x + y = 3$

$9^x.3^y = 3^4$

$\Rightarrow 3^{2x}.3^y = 3^4$

$\Rightarrow 2x + y = 4$ ....(ii)

By equation (i),

$x = 1$

From equation (i),

$1 + y = 3$

$\Rightarrow y = 2$

17. (b) $x = 3 + 2\sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{3 + 2\sqrt{2}}$

$= \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$

$= \frac{3 - 2\sqrt{2}}{9 - 8} = 3 - 2\sqrt{2}$

$\therefore \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$

$= 3 + 2\sqrt{2} + 3 - 2\sqrt{2} - 2 = 4$

$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = 2$

18. (a) $25a^2 + 40ab + 16b^2$

$= (5a + 4b)^2$

$= (5 \times 23 - 29 \times 4)^2$

$= (115 - 116)^2 = 1$

19. (a) $\frac{a}{3} = \frac{b}{2} \Rightarrow \frac{a}{b} = \frac{3}{2}$

$\therefore \frac{2a + 3b}{3a - 2b} = \frac{2 \times \frac{a}{b} + 3}{3 \times \frac{a}{b} - 2}$

$= \frac{2 \times \frac{3}{2} + 3}{3 \times \frac{3}{2} - 2} = \frac{6}{\frac{9 - 4}{2}} = \frac{12}{5}$

20. (b) Sum of x numbers = xy

Sum of y numbers = xy

$\therefore$ Required average

$= \frac{xy + xy}{x + y} = \frac{2xy}{x + y}$

21. (a) $x + \frac{1}{4x} = \frac{3}{2}$

$\Rightarrow 2x + \frac{1}{2x} = 3$

Cubing both sides,

$8x^3 + \frac{1}{8x^3} + 3 \times 2x \times \frac{1}{2x}$

$\left(2x + \frac{1}{2x}\right) = 27$

$\Rightarrow 8x^3 + \frac{1}{8x^3} + 3 \times 3 = 27$

$\Rightarrow 8x^3 + \frac{1}{8x^3} = 27 - 9 = 18$

22. (d) $x = \frac{4ab}{a + b} \Rightarrow \frac{x}{2a} = \frac{2b}{a + b}$

By componendo and dividendo,