Answers and Explanations

1. (b) \( \sec x = \csc y \)
   \[ \Rightarrow \cos x = \sin y \]
   \[ \Rightarrow \sin \left( \frac{\pi}{2} - x \right) = \sin y \]
   \[ \Rightarrow y = \frac{\pi}{2} - x \]
   \[ \Rightarrow x + y = \frac{\pi}{2} \]
   \[ \therefore \sin(x + y) = \sin \frac{\pi}{2} = 1 \]

5. (b) \( 7 \sin^2 \theta + 3 \cos^2 \theta = 4 \)
   \[ \Rightarrow 7 \frac{\sin^2 \theta}{\cos^2 \theta} + 3 = \frac{4}{\cos^2 \theta} = 4 \sec^2 \theta \]
   \[ \Rightarrow 7 \tan^2 \theta + 3 = 4(1 + \tan^2 \theta) \]
   \[ \Rightarrow 7 \tan^2 \theta + 4 \tan^2 \theta = 4 - 3 \]
   \[ \Rightarrow \tan^2 \theta = \frac{1}{3} \]
   \[ \Rightarrow \tan \theta = \frac{1}{\sqrt{3}} \]

6. (a) No. of terms in \( 1 + 5 + 9 + \ldots + 89 = n \)
   \[ \therefore a = 1 \]
   \[ d = 4 \]
   \[ a + (n - 1)d = 89 \]
   \[ 1 + (n - 1)4 = 89 \]
   \[ (n - 1)4 = 88 - 1 = 87 \]
   \[ n - 1 = 22 \]
   \[ n = 23 \]
   \[ \therefore \text{No. of terms} = \text{Now, } \sin^2 1^\circ + \sin^2 2^\circ + \sin^2 3^\circ + \sin^2 4^\circ + \ldots + \text{to 22 terms} + \sin^2 89^\circ \]
   \[ = (\sin^2 1^\circ + \cos^2 1^\circ) + (\sin^2 2^\circ + \cos^2 2^\circ) + \ldots \text{to 11 terms} + \left( \frac{1}{\sqrt{2}} \right)^2 \]
   \[ = 11 + \frac{1}{2} = 11 \frac{1}{2} \]

7. (c) \( \sec A - \cos A \)² + \( \csc A - \sin A \)² - \( \cot A - \tan A \)²
   \[ = \sec^2 A + \cos^2 A - 2 \sec A \cos A + \csc^2 A + \sin^2 A - 2 \csc A \sin A - \cot^2 A - \tan^2 A + 2 \cot A \tan A \]
   \[ = \sec^2 A - \tan^2 A + \cos^2 A + \sin^2 A + \csc^2 A - \cot^2 A - 2 \]
   \[ = 3 - 1 = 1 \]
CD = tree = h metre
AB = building = a metre
BC = ED = b metre

\[ \therefore \text{From } \Delta AED, \]
\[ \tan x = \frac{AE}{ED} \Rightarrow \tan x = \frac{a - h}{b} \]
\[ \Rightarrow b = (a - h) \cot x \quad \ldots(i) \]

\[ \text{From } \Delta ABC, \]
\[ \tan y = \frac{AB}{BC} \]
\[ \Rightarrow \tan y = \frac{a}{b} \]
\[ \Rightarrow b - a \cot y \quad \ldots(ii) \]

From equations (i) and (ii),
\[ (a - h) \cot x = a \cot y \]
\[ \Rightarrow a \cot x - h \cot x = a \cot y \]
\[ \Rightarrow h \cot x = a (\cot x - \cot y) \]
\[ \Rightarrow a = \frac{h \cot x}{\cot x - \cot y} \]

8. (a) \[ \cot 18^\circ \]
\[ \left( \cot 72^\circ \cdot \cos^2 22^\circ + \frac{1}{\tan 72^\circ \cdot \sec^2 68^\circ} \right) \]
\[ = \cot 18^\circ \cdot \cot 72^\circ \cdot \cos^2 22^\circ + \frac{\cot 18^\circ}{\tan 72^\circ \cdot \sec^2 68^\circ} \]
\[ = \cot 18^\circ \cdot \tan 18^\circ \cdot \cos^2 22^\circ + \frac{\cot 18^\circ}{\cot 18^\circ \cdot \cos^2 68^\circ} \]
\[ = \cos^2 22^\circ + \cos^2 68^\circ \]
\[ = \cos^2 22^\circ + \sin^2 22^\circ = 1 \]

\[ \therefore \tan(90^\circ - \theta) = \cot \theta; \]
\[ \sin(90^\circ - \theta) = \cos \theta; \]
\[ \sin^2 \theta + \cos^2 \theta = 1 \]

9. (b) \[ 2\sin^2 \theta + 3\cos^2 \theta \]
\[ = 2\sin^2 \theta + 2\cos^2 \theta + \cos^2 \theta \]
\[ = 2(\sin^2 \theta + \cos^2 \theta) + \cos^2 \theta \]
\[ = 2 + \cos^2 \theta \]
\[ \therefore \text{Minimum value of } \cos \theta = -1 \]
\[ \therefore \text{Required minimum value} = 2 + 1 + 3 \]

10. (c) \[ \frac{1}{\csc^2 51^\circ} + \sin^2 39^\circ + \tan^2 51^\circ \]
\[ = \frac{1}{\sin^2 51^\circ \cdot \sec^2 39^\circ} \]
\[ = \frac{\sin^2 51^\circ + \sin^2 39^\circ + \tan^2 (90^\circ - 39^\circ)}{\sin^2 (90^\circ - 39^\circ) \cdot \sec^2 39^\circ} \]
\[ = \frac{\cos^2 39^\circ + \sin^2 39^\circ + \cot^2 39^\circ}{\cos^2 39^\circ \cdot \sec^2 39^\circ} \]
\[ = \frac{1}{\cot^2 39^\circ \cdot \sec^2 39^\circ} \]
\[ = \sin(90^\circ - \theta) = \cos \theta, \tan(90^\circ - \theta) = \cot \theta \]
\[ = 1 = \cot^2 39^\circ - 1 \]
\[ = \csc^2 39^\circ - 1 = x^2 - 1 \]

11. (c) \[ \tan 4^\circ, \tan 43^\circ, \tan 47^\circ, \tan 86^\circ \]
\[ = \tan 4^\circ, \tan 43^\circ, \tan(90^\circ - 43^\circ), \tan(90^\circ - 4^\circ) \]
\[ = \tan 4^\circ, \tan 43^\circ, \cot 43^\circ, \cot 4^\circ = 1 \]
\[ \tan(90^\circ - \theta) = \cot \theta; \tan \theta \cdot \cot \theta = 1 \]
12. (b) \[ \frac{\tan \theta + \cot \theta}{\tan \theta - \cot \theta} = \frac{2}{1} \]
By componendo and dividendo,
\[ \frac{2 \tan \theta}{2 \cot \theta} = \frac{3}{1} \]
\[ \Rightarrow \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} = 3 \]
\[ \Rightarrow \sin^2 \theta = 3 \cos^2 \theta \]
\[ \Rightarrow \sin^2 \theta = 3 (1 - \sin^2 \theta) \]
\[ \Rightarrow 4 \sin^2 \theta = 3 \]
\[ \Rightarrow \sin^2 \theta = \frac{3}{4} \]
\[ \Rightarrow \sin \theta = \frac{\sqrt{3}}{2} \]

\[ \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{h+20} \Rightarrow \sqrt{3}h = h + 20 \]

\[ \Rightarrow (\sqrt{3} - 1)h = 20 \Rightarrow h = \frac{20}{\sqrt{3} - 1} \]
\[ = \frac{20}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \]
\[ = \frac{20(\sqrt{3} + 1)}{2} = 10(\sqrt{3} + 1) \text{ metre} \]

14. (b) When \( \theta = 0^\circ \)
\[ \sin^2 \theta + \cos^4 \theta = 1 \]
When \( \theta = 45^\circ \),
\[ \sin^2 \theta + \cos^4 \theta = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \]
When \( \theta = 30^\circ \)
\[ \sin^2 \theta + \cos^4 \theta = \frac{1}{4} + \frac{9}{16} = \frac{13}{16} \]

15. (d) \( \sin \theta + \csc \theta = 2 \)
\[ \Rightarrow \sin \theta + \frac{1}{\sin \theta} = 2 \]
\[ \Rightarrow \sin^2 \theta - 2 \sin \theta + 1 = 0 \]
\[ \Rightarrow (\sin \theta - 1)^2 = 0 \Rightarrow \sin \theta = 1 \]
\[ \therefore \sin^5 \theta + \csc^5 \theta = 1 + 1 = 2 \]

16. (d) \( \sin \theta = \cos (90^\circ - \theta); \)
\[ \sin (90^\circ - \theta) = \cos \theta \]
\[ \therefore \sin 85^\circ = \sin (90^\circ - 5^\circ) = \cos 5^\circ \]
\[ \therefore (\sin^2 5^\circ + \sin^2 85^\circ) + (\sin^2 10^\circ + \sin^2 80^\circ) + \ldots \]
\[ \text{to terms} + \sin^2 45^\circ + \sin^2 90^\circ \]
\[ = 8 \times 1 + \frac{1}{2} + 1 = 9 \frac{1}{2} \]

17. (b)
20. (c) \[
\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = 3
\]
\[
\Rightarrow \sin \theta + \cos \theta = 3 \sin \theta - 3 \cos \theta
\]
\[
\Rightarrow 4 \sin \theta = 2 \sin \theta \Rightarrow \tan \theta = 2
\]
\[
\therefore \sin^4 \theta - \cos^4 \theta = (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta)
\]
\[
= \sin^2 \theta - \cos^2 \theta
\]
\[
= \cos^2 \theta (\tan^2 \theta - 1)
\]
\[
\Rightarrow \frac{\tan^2 \theta - 1}{1 + \tan^2 \theta} = \frac{4 - 1}{1 + 4} = \frac{3}{5}
\]

21. (c) \((\tan 1^\circ, \tan 89^\circ), (\tan 2^\circ, \tan 88^\circ), \ldots, (\tan 45^\circ, \tan 45^\circ)\)
\[
= (\tan 1^\circ, \cot 1^\circ), (\tan 2^\circ, \cot 2^\circ), \ldots, 1
\]
\[
\therefore \begin{bmatrix} \tan(90^\circ - \theta) = \cot \theta \\ \cot(90^\circ - \theta) = \tan \theta, \tan \theta, \cot \theta = 1 \end{bmatrix}
\]

22. (a) \[
AB = CD = h \text{ metre (Height of pole)}
\]
From $\triangle ABE$, 
\[
\tan 30^\circ = \frac{h}{x}
\]
\[
\frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow \sqrt{3}h = x \quad \ldots (i)
\]
From $\triangle DBC$, 
\[
\tan 60^\circ = \frac{h}{100 - x}
\]
\[
\Rightarrow \sqrt{3} = \frac{h}{100 - x}
\]
\[
\Rightarrow \sqrt{3} (100 - x) = h
\]
\[
\Rightarrow \sqrt{3} (100 - \sqrt{3}h) = h 
\]
[From equation (i)]
\[
\Rightarrow 100\sqrt{3} - 3h = h \Rightarrow 4h = 100\sqrt{3}
\]
\[
\Rightarrow h = 25\sqrt{3} \text{ metre}
\]
23. (a) $\sec^2 \theta + \tan^2 \theta = 7$
\[
\Rightarrow 1 + \tan^2 \theta = 7 - 1 = 6
\]
\[
\Rightarrow \tan^2 \theta = 6 \Rightarrow \tan \theta = \sqrt{3}
\]
\[
\Rightarrow \theta = 60^\circ
\]
24. (d) 
\[
\frac{\sin 39^\circ}{\cos 51^\circ} + 2 \tan 11^\circ \cdot \tan 79^\circ \cdot \tan 31^\circ \cdot \tan 59^\circ \cdot \tan 45^\circ
\]
\[
= \frac{\sin 39^\circ}{\cos (90^\circ - 39^\circ)} + 2 \tan 11^\circ.
\]
\[
\tan (90^\circ - 11^\circ) \cdot \tan 31^\circ \cdot \tan (90^\circ - 59^\circ)
\]
\[
= \sin 39^\circ + 2 \tan 11^\circ \cdot \cot 11^\circ \cdot \tan 31^\circ \cdot \cot 31^\circ
\]
\[
= 3(\sin^2 21^\circ + \sin^2 69^\circ)
\]
\[
= \frac{\cos^2 \theta}{\cot^2 \theta - \cos^2 \theta} = 3
\]
\[
\Rightarrow \cos^2 \theta = 3 \cot^2 \theta - 3 \cos^2 \theta
\]
\[
\Rightarrow 4 \cos^2 \theta = 3 \cot^2 \theta = \frac{3 \cos^2 \theta}{\sin^2 \theta}
\]
\[
\Rightarrow 4 \cos^2 \theta - 3 \frac{\cos^2 \theta}{\sin^2 \theta} = 0
\]
\[ \Rightarrow \cos^2 \theta \left( 4 - \frac{3}{\sin^2 \theta} \right) = 0 \]

\[ \therefore 4 - \frac{3}{\sin^2 \theta} = 0 \]

\[ \Rightarrow 4 \sin^2 \theta = 3 \]

\[ \Rightarrow \sin \theta = \frac{\sqrt{3}}{2} \times \sin 60^\circ \]

\[ \Rightarrow \theta = 60^\circ \]

\[ \cos^2 \theta = 0 \Rightarrow \theta = 90^\circ \]

27. (c) \[ A = \tan 11^\circ \tan 29^\circ \]
\[ B = 2 \cot 61^\circ \cot 79^\circ \]
\[ = 2 \cot(90^\circ - 29^\circ) \cot(90^\circ - 11^\circ) \]
\[ = 2 \tan 29^\circ \tan 11^\circ \quad [\therefore \cot(90^\circ - \theta) = \tan \theta] \]
\[ = 2A \]

28. (b) \[ \cos^2 \alpha + \cos^2 \beta = 2 \]
\[ \Rightarrow 1 - \sin^2 \alpha + 1 - \sin^2 \beta = 2 \]
\[ \Rightarrow \sin^2 \alpha + \sin^2 \beta = 0 \]
\[ \Rightarrow \sin \alpha = \sin \beta = 0 \]
\[ \Rightarrow \alpha = \beta = 0 \]
\[ \therefore \tan^3 \alpha + \sin^5 \beta = 0 \]

29. (b) \[ \tan(2\theta + 45^\circ) = \cot 3\theta \]
\[ = \tan(90^\circ - 3\theta) \]
\[ \Rightarrow 2\theta + 45^\circ = 90^\circ - 3\theta \]
\[ \Rightarrow 5\theta = 90^\circ - 45^\circ = 45^\circ \]
\[ \therefore \theta = 9^\circ \]

30. (b) \[ AB = \text{Length of the thread} = 150 \text{ metre} \]
\[ \angle BAC = 60^\circ \]

31. (b) \[ \cos \theta = \frac{15}{17} \]
\[ \Rightarrow \sec \theta = \frac{1}{\cos \theta} = \frac{17}{15} \]
\[ \therefore \cot (90^\circ - \theta) = \tan \theta \]
\[ = \sqrt{\sec^2 \theta - 1} \]
\[ = \sqrt{\frac{17^2}{15^2} - 1} = \frac{\sqrt{289}}{225} - 1 \]
\[ = \frac{\sqrt{289 - 225}}{225} = \frac{64}{225} = \frac{8}{15} \]

32. (a) \[ \sec^2 \theta - \tan^2 \theta = 1 \]
\[ \sec^2 \theta + \tan^2 \theta = \frac{7}{12} \]
\[ \therefore \sec^4 \theta - \tan^4 \theta \]
\[ = (\sec^2 \theta - \tan^2 \theta)(\sec^2 \theta + \tan^2 \theta) \]

33. (a) \[ \cos x + \cos y = 2 \]
\[ \therefore \cos x \leq 1 \]
\[ \Rightarrow \cos x = 1; \cos y = 1 \]
\[ \Rightarrow x = y = 0^\circ \]
\[ \sin x = \sin y = 0 \]

34. (a) \[ \tan 15^\circ \cdot \cot 75^\circ + \tan 75^\circ \cdot \cot 15^\circ \]
\[ = \tan 15^\circ \cdot \cot (90^\circ - 15^\circ) + \tan (90^\circ - 15^\circ) \cdot \cot 15^\circ \]
\[ = \tan^2 15^\circ + \cot^2 15^\circ \]
\[ = \frac{1}{2 - \sqrt{3}} \left[ \cot (90^\circ - \theta) = \cot \theta \\ \cot (90^\circ - \theta) = \tan \theta \right] \]
\[ \tan 15^\circ = 2 - \sqrt{3} \]
\[ \therefore \ \cot 15^\circ \]
\[ = \frac{1}{2 - \sqrt{3}} = \frac{2 + \sqrt{3}}{(2 - \sqrt{3})(2 + \sqrt{3})} \]
\[ = 2 + \sqrt{3} \]
\[ \therefore \ \tan^2 15^\circ + \cot^2 15^\circ \]
\[ = (2 - \sqrt{3})^2 + (\sqrt{2} + \sqrt{3})^2 \]
\[ = 2(4 + 3) = 14 \]

35. (b) \[ \tan \theta + \cot \theta = 2 \]
\[ \Rightarrow \ \tan \theta + \frac{1}{\tan \theta} = 2 \]
\[ \Rightarrow \ \tan^2 \theta + 1 = 2 \tan \theta \]
\[ \Rightarrow \ \tan^2 \theta - 2 \tan \theta + 1 = 0 \]
\[ \Rightarrow \ (\tan \theta - 1)^2 = 0 \]
\[ \Rightarrow \ \tan \theta = 1 \]
\[ \Rightarrow \ \cot \theta = 1 \]
\[ \therefore \ \tan^5 \theta + \cot^4 \theta = 1 + 1 = 2 \]

36. (c)

Let \( PQ = h \) metre and \( BQ = x \) metre.

From \( \triangle APQ \),
\[ \tan 30^\circ = \frac{h}{x + 20} \]
\[ \Rightarrow \ \frac{1}{\sqrt{3}} + \frac{h}{x + 20} \]
\[ \Rightarrow \ \sqrt{3}h = x + 20 \] \[ \text{(i)} \]

From \( \triangle PQB \),
\[ \tan 60^\circ = \frac{PQ}{BQ} = \frac{h}{x} \]
\[ \Rightarrow \ \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x \]
\[ \Rightarrow \ x = \frac{h}{\sqrt{3}} \] \[ \text{(ii)} \]

\[ \therefore \ \sqrt{3}h = \frac{1}{\sqrt{3}}h + 20 \]

[From equation (i) and (ii)]
\[ \Rightarrow \ 3h - h = 20 \sqrt{3} \]
\[ \Rightarrow \ 2h = 20 \sqrt{3} \]
\[ \therefore \ h = 10 \sqrt{3} \] metre

37. (a) \[ \sin \theta - \cos \theta = \frac{7}{13} \] \[ \text{(i)} \]
\[ \sin \theta + \cos \theta = x \] \[ \text{(ii)} \]

On squaring both equations and adding,
\(2(\sin^2 \theta + \cos^2 \theta) = \frac{49}{169} + x^2\)

\[\Rightarrow x^2 = 2 - \frac{49}{169} = \frac{338 - 49}{169}\]

\[= \frac{289}{169} \Rightarrow x = \frac{17}{13}\]

\[\Rightarrow \sin^2 \theta - 2 \sin \theta + 1 = 0\]

\[\Rightarrow (\sin \theta - 1) = 0\]

\[\Rightarrow \sin \theta = 1 \Rightarrow \cosec \theta = 1\]

\[\therefore \sin^{100} \theta + \cosec^{100} \theta = 1 + 1 = 2\]

38. (d) \(\sin (2x - 20^\circ) = \cos (2y + 20^\circ)\)

\[\Rightarrow \sin (2x - 20^\circ) = \sin (90^\circ - 2y - 20^\circ)\]

\[\Rightarrow 2x - 20^\circ = 70^\circ - 2y\]

\[\Rightarrow 2x + 2y = 70 + 20 - 90^\circ\]

\[\Rightarrow x + y = 45^\circ\]

\[\therefore \tan (x + y) = \tan 45^\circ = 1\]

39. (b) Let the number of terms be \(n\), then

By \(t_n = a + (n-1)d\) \(85 = 5 + (n-1)\)

\[\Rightarrow n - 1 = 85 - 5 = 80\]

\[\Rightarrow n = 81\]

\[\therefore \sin^2 5^\circ + \sin^2 6^\circ + \ldots + \sin^2 45^\circ + \ldots + \sin^2 85^\circ = \sin^2 84^\circ + \sin^2 84^\circ + \ldots + \text{to 40 terms} + \sin^2 45^\circ\]

\[= (\sin^2 5^\circ \sin^2 85^\circ) + (\sin^2 6^\circ + \ldots + \sin^2 84^\circ) + \ldots + \text{to 40 terms} + \sin^2 45^\circ\]

\[= (\sin^2 5^\circ + \cos^2 5^\circ) + (\sin^2 6^\circ + \ldots + \cos^2 6^\circ + \ldots + \text{to 40 terms} + \sin^2 45^\circ\)

\[\sin (90^\circ - \theta) = \cos \theta\]

\[\sin^2 \theta + \cos^2 \theta = 1\]

\[= 40 + \frac{1}{2} = 40 \frac{1}{2}\]

40. (b) \(\sin \theta + \cosec \theta = 2\)

\[\Rightarrow \sin \theta + \frac{1}{\sin \theta} = 2\]